International Journal of Bioelectromagnetism
Vol. 9 No. 2 2007

Reconstruction of 3-dimensional distribution of tangential current density components from magnetic vector measurement

Chikayoshi Sumi, Keita Hayakawa and Naoki Koike
Dept of Electrical & Electronics Engineering, Faculty of Sci & Tech, Sophia University
c-sumi@sophia.ac.jp

Abstract

A current density reconstruction method which enables 3-dimensional (3D) electric property reconstruction by extending our previously proposed 2D approach is described. On the basis of Biot-Severt's law, a 3D distribution of tangential current density components can be reconstructed when the surface of the target body is widely flat or spherical. Several stabilization methods are also described.

1. Introduction

For living *in vivo* tissues, we have been developing noninvasive electromagnetic techniques to reconstruct the internal distributions of conductivity and permittivity. In one of our approaches [1], on the basis of the magnetic vector measurements performed in the vicinity of the target, the internal current density vector distribution is reconstructed (an inverse problem of Biot-Severt's law [2]), subsequently from which such tissue electric properties are reconstructed. Our techniques enable us to evaluate the electric conductive paths of normal and cultured nerves as well as tissue electric properties (brain, heart, muscle and so forth). Occasionally such properties also express functions that are determined by physiological or pathological states. The nondestructive evaluations of structures (e.g., electric circuits) and materials can also be performed.

Thus far, we have reported for a two-dimensional (2D) medium a reconstruction method for a 2D conductivity distribution using a 2D distribution of a 2D current density vector [1]. Only the tangent components of a magnetic vector are used to reconstruct the 2D current measurement. Although such a 2D current measurement is often performed on a 3-dimensional (3D) body, e.g., on a human chest for diagnosis of a heart (see references in ref. [1]), the spatial resolution in the depth direction cannot be obtained. This limits the application of our conductivity reconstruction. That is, to enable such a conductivity reconstruction on a 3D target, a measurement of a 3D distribution of either full three or two current density vector components must be realized [1]. However, as is well known, a 3D current vector cannot be evaluated from a full set of 3D magnetic vector components [2].

Thus, in this report, we propose a novel approach in that for a 3D target only two tangential components of a 3D current vector are reconstructed, i.e., a new inverse problem of Biot-Severt's law. That is, *a 3D distribution of two tangential current density components* is a reconstruction target. This is also motivated by the fact that the normal current density component does not contribute to the magnetic vector outside of the body when the surface of the target body is widely flat or spherical.

For the current density reconstruction, our previously developed regularization for stabilizing a 2D current density vector reconstruction [1] is also used together with our designed weighting functions similarly to LORETA [3]. By properly weighting in the depth direction the penalty terms (i.e., identity and Laplacian matrices) *for the respective current density components*, the accuracy of the reconstruction increases. Moreover, we also introduce a lifting procedure for three or tangential two directions' pickup coils during the measurements of a 3D or 2D magnetic vector. Although such measurements decrease the signal-to-noise ratio (SNR) of the magnetic measurement, the measurement performed to a certain height increases the information about the target tangential current density distributions. A 3D array of such pickup coils can also be used instead.

2. Reconstruction of tangential current density components

On the basis of Biot-Severt's law, simultaneous equations are obtained for a 3D distribution of the two tangential current density vector components **J**, i.e.,

$$\mathbf{B} = \mathbf{R}\,\mathbf{J}, \qquad (1)$$

where **B** denotes a 3D distribution of a 3D or tangential 2D magnetic vector measured by the procedures of scanning and lifting of pickup coils or the use of a 3D array of pickup coils. **R** is a lead matrix.

The 3D distribution of tangent current density components is estimated using a regularized least-squares method as in our previous 2D current reconstruction [1]. Regarding the regularization, we newly introduce two new ideas. The first one is regularization is properly performed on the respective current density components on the basis of the measurement accuracies of the correspondingly contributed magnetic vector components. For instance, the respective regularization parameters are set to be proportional to the evaluated variances. The second one is the regularization is properly performed at each position similarly to LORETA using the weighting functions that are expressed by the magnitude of a lead field or a Green function. However, in our regularization, the respective current density components are properly regularized using the weighting functions that are expressed by the corresponding components of the lead field or Green function. This increases the effectiveness of the regularization in that the stability and accuracy (compensation) of the reconstruction increase.

The magnetic measurement data should be properly low-pass-filtered in advance to reduce measurement noise and when real-time evaluation is not required, an ensemble or additional average should be implemented on the measured raw data. When a whole conductive target is not dealt with (i.e., a finite region of interest (ROI) is set in the target body), our previously developed windowing [1] should be performed in advance to remove the magnetic data generated by the current density outside of the ROI.

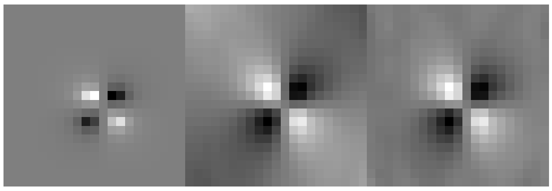
3. Simulation

The simulated target used is a rectangular parallelepiped 3 mm in height (z), with 40 mm lateral (x) and elevational (y) widths. A steady current distribution generated by a uniform unit voltage applied in the lateral (x) direction was observed. The target had a cubic inclusion with 2 mm sides having a twofold conductivity such that the surface thereof appeared at the center of an x-y surface of the target. A rectangular parallelepiped with dimensions of 20 (x), 20 (y), and 3 (z) mm was set in the target as a ROI such that it included the cubic inclusion at the center. The 3D distribution of a magnetic vector was measured at a height of 0.2 mm from the upper surface of the ROI. Sampling was performed at steps of 1 mm both for the magnetic vectors and current densities.

Table 1 shows the SNRs of the reconstructed current densities Jx (lateral) and Jy (elevational) evaluated at each depth. As shown, the SNRs of the nonregularized case (shown in parentheses) are low in the deep region, particularly those of Jy. Fig. 1 shows the normalized gray images of the reconstructions Jy obtained with regularization and no regularization at a depth of 2 mm as well as that of an original. As shown, the larger the depth, the lower the spatial resolutions of both reconstructions Jy. However, regularization significantly increases the SNRs of Jy. The difference in SNR between the regularized and nonregularized Jy is due to the fact that the magnitudes of nonregularized Jy were estimated to be smaller than those of regularized Jy. Although the large setting of the regularization parameters for Jy increased the accuracy of the reconstruction value, the SNRs did not increase due to the degradation of the spatial resolutions.

Table 1. SNRs by regularization (and no regularization) [dB]

Depth (mm)	Jx	Jy
0	52.5 (29.2)	26.9 (3.0)
1	24.1 (13.6)	0.8 (-17.8)
2	21.8 (10.6)	3.7 (-20.2)



(a) (b) (c)

Fig. 1. Normalized Jy: (a) Original, (b) Nonregularized, (c) Regularized.

4. Conclusions

The methods described will enable our proposed 3D electric property reconstruction. In the near future, we will report the effects on the electric property reconstruction of current reconstruction errors together with the effects on the current reconstruction accuracies of the dimension of a measured magnetic vector (i.e., 3D or 2D), the height of the 3D distribution of the magnetic vector data and the height of a target, and the sampling steps of magnetic vectors and current densities.

References

[1] C. Sumi, K. Nagumo, “Simulation study on reconstruction of electric conductivity distribution from magnetic vector field measurement,” *Meas Sci Tech*, 2004, pp. 1239-1250.
[2] B. Roth, N. G. Sepulveda, J. P. Wikswo, “Using a magnetometer to image a two-dimensional current distribution,” *J Appl Phys*, 1989, pp. 361-372.
[3] R. D. Pascual-Marqui, “Review of methods for solving the EEG inverse problem,” *IJBEM*, 1999, pp. 75-86.