International Journal of Bioelectromagnetism
Vol. 9 No. 2 2007

Mathematical expressions of reconstructions of conductivity and permittivity from current density

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Abstract

Mathematical expressions of reconstructions of conductivity and permittivity using measured current density are described. The current density is measured electromagnetically. By placing a pickup coil close to the target, these electric property reconstructions can be obtained. The reconstructions can be performed with respect to a target in situ as well, e.g., nerve circuits and various tissues (brain, heart, muscle and so forth).

1. Introduction

For living *in vivo* tissues, we have been developing noninvasive electromagnetic techniques for reconstructing the internal distributions of conductivity and permittivity. In our one approach [1], on the basis of the magnetic vector measurements performed in the vicinity of the target, the internal current density vector distribution is reconstructed (an inverse problem of Biot-Severt's law [2]), subsequently from which such tissue electric properties are reconstructed. In ref. [1], a steady case is dealt with, whereas in this report, a dynamic case is dealt with. That is, in ref. [1], only conductivity is reconstructed, whereas in this report, both conductivity and permittivity are our targets.

For conductivity reconstruction, for instance, in ref. [3], paired excitation and detection electrodes are used; in ref. [4], paired excitation coils and detection electrodes are used; and in ref. [5], paired excitation and detection coils are used. Permittivity reconstruction is also reported in ref. [6]. In all these reconstructions, a sensitivity theorem [7] is used and then, the reconstruction problems fall in a nonlinear-integration-type inverse problem.

However, the inverse problems in our approach belong to a linear-differential-type problem [8]. Our approach enables reconstructions in situ. That is, a target in which current normally exists can also be dealt with only by placing pickup coil close to the target. Thus, our techniques enable us to evaluate the electric conductive paths of normal and cultured nerves as well as tissue electric properties (brain, heart, muscle and so forth). Occasionally such properties also express functions that are determined by physiological or pathological states. Nondestructive evaluations of structures (e.g., electric circuit) and materials can also be performed.

Thus far, we have reported for a two-dimensional (2D) medium a reconstruction method of a 2D conductivity distribution using a 2D distribution of a 2D current density vector [1]. To enable such a reconstruction on a 3D target, a measurement of a 3D distribution of either full three or two current density vector components must be realized [1]. However, as is well known, a 3D current vector cannot be evaluated from a full set of 3D magnetic vector components [2]. Thus, in ref. [3], we proposed a novel approach in that for a 3D target only two tangential components of a 3D current vector are reconstructed, i.e., a new inverse problem of Biot-Severt's law. This was also motivated by the fact that the normal current density component does not contribute to the magnetic vector outside of the body when the surface of the target body is widely flat or spherical. Moreover, we also introduced a lifting procedure for pickup coils to realize a measurement of a 3D distribution of a 3D or tangential 2D magnetic vector.

2. Mathematical expressions

A. Review of conductivity reconstruction

In a steady current field, if no current source exists in a region of interest (ROI), the constitutive equation and governing equation are respectively,

$$\mathbf{J} = \sigma \mathbf{E} \qquad (1)$$

$$\nabla \times \mathbf{E} = \mathbf{0}, \qquad (2)$$

where **J** and **E** denote respectively current density and electric field, and σ denotes conductivity. By eliminating **E** by substituting (1) into (2), simultaneous first order partial differential equations (PDEs) regarding σ are obtained.

$$\sigma\nabla\times\mathbf{J}-\nabla\sigma\times\mathbf{J}=\mathbf{0} \quad (3)$$

The PDEs can also be expressed as

$$\nabla\times\mathbf{J}-\nabla(\ln\sigma)\times\mathbf{J}=\mathbf{0}\,. \quad (4)$$

The spatial inhomogeneous coefficients are derived from measured current densities in the ROI. Numerical solutions are specifically described in refs. [9,10]. As mentioned above, measurement of only two current density components also enables the reconstruction by using reference conductivities in the ROI.

When realizing a reference point in the ROI [9], if the reference conductivity is known, the absolute conductivity distribution can be reconstructed, whereas if not known, by assigning an arbitrary value (e.g., unit) at an arbitrary point in the ROI, the relative conductivity distribution can be reconstructed. In this case, however, measurements of more than two independent current density distributions generated by using different positions of current sources injected or induced or using some attachment are required. When realizing a reference region (i.e., not a point) in the ROI such that the reference region widely extends in the direction of the generated current flow [10], measurement of only one current density distribution is required. By depicting the characteristic curves of the PDEs (i.e., using measured **J**), we can obtain such a meaningful information about the proper configurations of current sources and references.

B. Conductivity and permittivity reconstruction

In a dynamic current field, if no current source exists in an ROI, the constitutive equation and governing equation are respectively,

$$\mathbf{J}=\sigma\mathbf{E}+\frac{\partial}{\partial t}(\varepsilon\mathbf{E}) \quad (5)$$

$$\nabla\times\mathbf{E}=-\frac{\partial}{\partial t}(\mathbf{B})\,, \quad (6)$$

where **B** denotes a magnetic vector and ε denotes permittivity. By eliminating **E**, simultaneous first-order PDEs regarding σ and ε are obtained.

$$\nabla\times[(A-jC)\mathbf{J}]=-\frac{\partial}{\partial t}(\mathbf{B})$$

$$\text{where } A=\sigma/[\sigma^2+(2\pi f)^2\varepsilon^2],$$

$$C=(2\pi f\varepsilon)/[\sigma^2+(2\pi f)^2\varepsilon^2]\,, \quad (7)$$

where j denotes an imaginary unit, both **J** and **B** denote, in an analytic form, current density vector and magnetic vector, and f denotes an instantaneous frequency of **J**. The magnetic vector **B** can be obtained by Biot-Severt's law using the measured **J**. As is well known, the permeability of tissue is almost the same as that of air. By solving eq. (7) using references σ and ε, the distributions of σ and ε can be reconstructed. The references should be realized as a reference region like that of conductivity in a steady case. If such a proper configuration cannot be realized, measurements of more than two independent field signals **J** generated at different times or by using different positions of current sources or some attachment are required.

When **J** is simply sinusoidal, a single frequency is used as f in eq. (7) instead of the instantaneous frequency. Occasionally, only an arbitrary frequency component in signals may also be used approximately. By these or signal analyses of σ and ε, frequency variances may also be evaluated.

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