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The inverse problem of electrocardiography: industrial solutions and simulations

Eric J. Voth
Endocardial Solutions (AF Division, St. Jude Medical), St. Paul, MN, USA

Abstract—**A common concern in solving the inverse problem of electrocardiography is how to efficiently assess the accuracy of the reconstruction algorithms under a wide variety of clinically relevant conditions. This paper describes a research testbed based on the SCIRun/ BioPSE computational steering software that allows rapid prototyping and evaluation of potential new algorithm improvements, as well as comparisons to Endocardial Solutions' clinical diagnostic system, which solves the inverse problem using its own high-order boundary element method and regularization scheme.**

Keywords—**BioPSE, electrocardiography, ESI, inverse problem, non-contact mapping, SCIRun, simulation**

I. Introduction

The inverse problem of electrocardiography is usually formulated as estimating epicardial source potentials from measurements made by body surface electrodes [1]. But it can also be solved to determine endocardial sources from voltage data on an intracardiac probe or balloon [2], [3]. Both alternatives are mathematically ill-posed, because all of the known data (voltage and normal current, which is zero on the body or the balloon) lie on only one of the two boundary surfaces of the domain. The latter formulation, however, has the advantage that its domain is homogeneous blood, instead of the differing and anisotropic conductivities of torso muscles, lungs, fat, and bones. The positions of the measurement electrodes are also typically known with greater precision that those of a body-surface electrode net.

The EnSite™ system [3]–[6] is comprised of a specialized 3mm-diameter catheter, a Patient Interface Unit (PIU) containing custom amplifiers and hardware, and analysis software running on a dual-Xeon Linux workstation. Data can be recorded to DVD and reviewed at will. The EnSite™ catheter is typically inserted through a femoral artery or vein and navigated to the desired chamber of the heart, where the 18mm-diameter balloon is deployed (Fig. 1). An 8x8 mesh of laser-etched wire electrodes measures the potential field on its surface at 1200 Hz; the signals are amplified by the PIU and sent over fiber-optic Ethernet to the workstation. Other EP catheters connected to the EnSite system can be located relative to the balloon by means of independent 5 KHz electrical signals fed into a proprietary algorithm (validated in [4]–[5]). These catheters can be steered within the chamber to build up a geometry model of the endocardial surface relative to the balloon position. And the balloon surface is modeled based on its deployed electrode positions, which are calibrated during manufacturing.

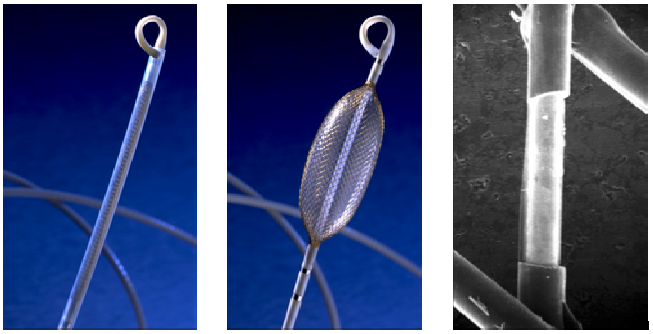

Fig. 1. The EnSite™ catheter: sheathed, deployed, and a magnified view of one wire electrode

Within the chamber, the electric field generated by myocardial muscle cells is well modeled by the Laplace equation $\nabla^2 v = 0$. The endocardial and balloon surfaces are represented by tensor-product bicubic splines, and a high-order boundary element method (BEM) solves the Laplace equation using Green's Second Formula

$$\iint_{\partial D} \left[v\frac{\partial w}{\partial n} - w\frac{\partial v}{\partial n} \right] dA = \iiint_{D} \left[v\nabla^2 w - w\nabla^2 v \right] dD$$

as in Equation (40) of [1]. A proprietary enhancement of zero-order Tikhonov regularization provides more accurate reconstructions of the endocardial potential field. The system is currently in operation in about 450 hospitals and clinics in 30 countries, and has been used in approximately 20,000 patient cases since its 1998 introduction.

The SCIRun Bioelectric Problem-Solving Environment (BioPSE) [7]–[8] is an open-source software package for computational simulation and visualization of bioelectric field problems. Users program it by graphically arranging a network of "modules," each performing a specific task, and connecting them with color-coded "pipes" to describe the flow of various data objects between them. The package can be extended and customized with user-written modules and interfaces in C++ and Tcl, respectively.

This paper describes a research extension of SCIRun/ BioPSE that incorporates the algorithms from ESI's clinical product. Starting from a known endocardial activation pattern, BioPSE's linear boundary element method solves the forward problem to model the voltages that would be measured by the EnSite™ catheter electrodes, and ESI's own high-order BEM solves the inverse problem. Maps and individual waveforms from the inverse solution can be compared to the initial data, and potential improvements to the BEM, geometry modeling, or regularization algorithms can quickly be prototyped and tested.

II. METHODS

The SCIRun workspace (Fig. 2) contains two distinct networks. The two columns on the right simulate an activation wave on the left atrial endocardial model. The three columns on the left form the main analysis network. (The six slightly larger modules are "subnets", each an encapsulation of several related modules that allow a little more clarity in the top-level view. There would be 46 modules in the main network without the use of subnets.)

The main network reads surface models of the balloon and endocardium and a matrix of time data on the endocardium, solves the forward problem, pipes it through ESI's SplineBEM inverse solver, and sends the results and the original data to various visualization subnets. The single Viewer module can spawn many OpenGL windows, each of which can display a customized subset of the data, and all of which can be rotated in unison if desired.

The endocardial surface model (Fig. 4) was segmented from a CT scan of a human left atrium and converted into SCIRun format, as a mesh with 4362 triangular elements. An artificial "activation pattern" $v(t - t_0)$ was generated at each node, where v approximates a typical extracellular potential and t_0 is proportional to the distance to an arbitrary node near the right atrium where the activation was assigned to begin. (In the near future we look forward to the fully graphical SCIRun integration of the CardioWave simulation package [9]–[10] developed at Duke University. Then we will be able to generate realistic activation patterns, even including local regions of ischemia or conduction block, and see how well they are reconstructed.)

For consistency, the EnSite model for the linear forward solver (1322 nodes, 2640 elements) was triangulated from the bicubic spline model used in the inverse problem. This removes one potential source of error to the maximum extent possible. Otherwise the forward and inverse BEM solvers are completely independent, so as to avoid biasing the results by committing "inverse crimes". The mesh densities for the forward problem are much finer than those for the inverse problem, in order to begin with a high-quality forward solution. But the differences between the original and the reconstructed endocardial waveforms are caused by discretization errors in both solvers, as well as the fundamental ill-posedness of the problem.

III. RESULTS

Figs. 3 and 4 show anterior views of the "simulated" atrial activation and the ESI reconstruction (respectively) at one time instant, as well as the 20 points on the endocardial surface at which waveforms were sampled. The inset shows the potential on the balloon surface, in the same color scale. The regularization parameter was 1.0; no attempt was made to optimize this or any other parameters of the inverse solver. Fig. 5 shows pairs of simulated (solid) and reconstructed (dashed) waveforms from six of the numbered points, along with the normalized cross correlations of each pair. (Notice that all the reference waveforms are identical, only shifted in time.) Over all 20 sample points, the mean cross correlation was 0.876±0.087. As is known in the literature [11], the magnitude of the computed potentials is attentuated by any regularization scheme, especially in regions farther from the measuring electrodes (e.g. the inferior region, points 16–19).

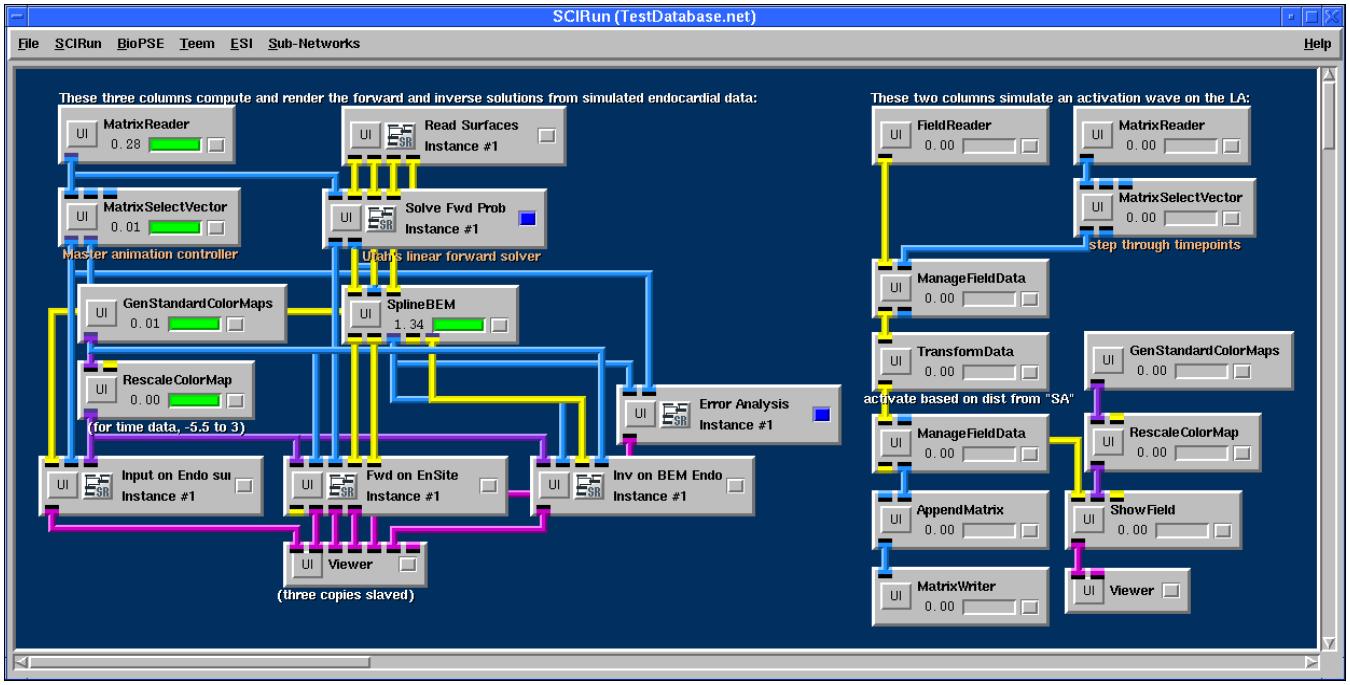

Fig. 2. The "TestDatabase" network in BioPSE. Execution generally proceeds from top to bottom, for readability.

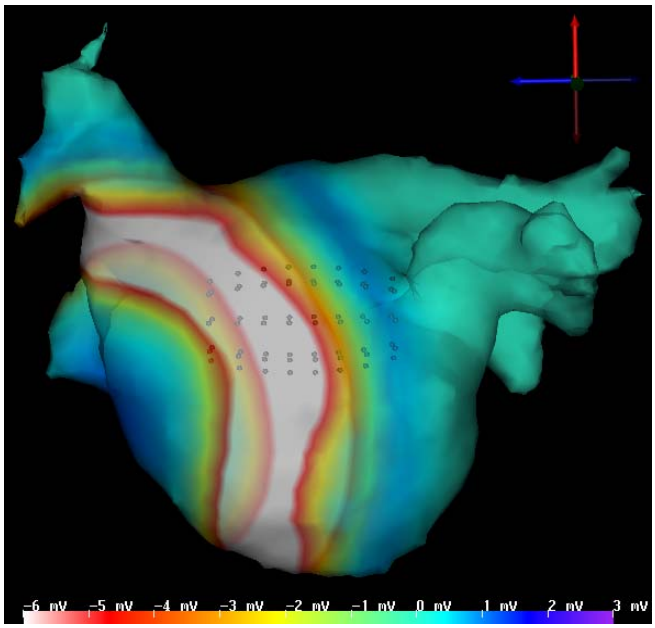

Fig. 3. Translucent anterior view of human LA during simulated activation (left atrial appendage at foreground right). EnSite™ electrode positions visible in interior.

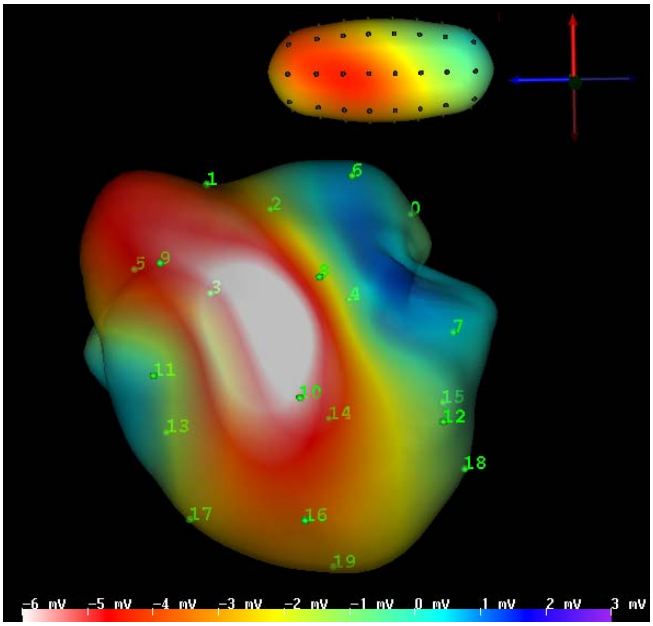

Fig. 4. Bicubic spline surface for inverse problem (the pulmonary veins and appendage are ignored) showing nodes at which waveforms were sampled and analyzed. Fully obscured node points (0–5, 7, 13–15) are on the posterior side. Inset shows voltage on balloon surface.

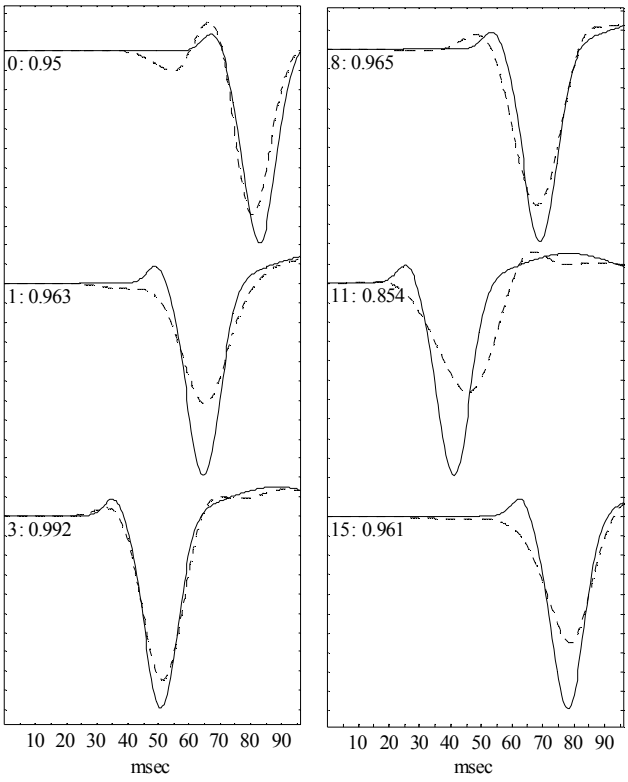

Fig. 5. Waveforms and cross correlations at six endocardial sample points (solid = input, dashed = inverse solution). Vertical scale is in millivolts, though this is arbitrary.

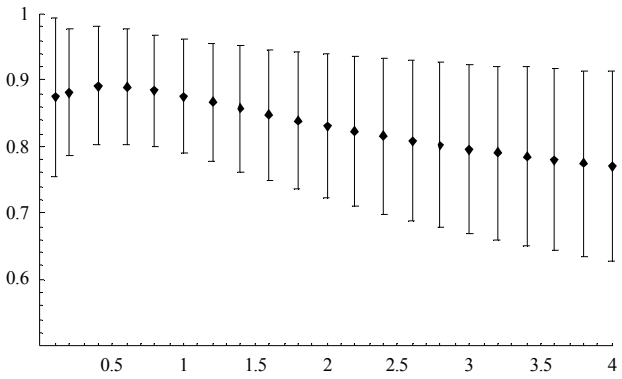

Fig. 6. Mean cross correlation of the 20 waveform pairs as a function of regularization parameter.

(Note that these data do not illustrate typical performance of the EnSite™ system. They are simulated, to illustrate the capabilities of this automated analysis framework.)

Since the SCIRun command line can run Tcl scripts, it is straightforward to execute automatically any or all of the network multiple times with different parameters. For example, Fig. 6 plots the mean cross correlation of these 20 waveforms as a function of regularization parameter, which varies from 0.1 to 4.0. As is also well known, regularization parameters either too high or too low produce suboptimal reconstructions.

IV. Discussion

This paper illustrates the flexibility and usefulness of the SCIRun/BioPSE framework for industrial research. Our plan at ESI is to assemble a significant database of actual human cardiac models from various chambers, clinically relevant balloon positions and waveform sample points, and high-quality endocardial activation simulations in various rhythms and arrythmias. Then we will be able to automatically evaluate many of the new methods suggested in the recent literature [12]–[15] for improving the inverse solution, to the ultimate benefit of the patients treated by the EnSite™ system.

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