



Adaptive Growth and Optimization of Coronary Arterial Tree Models

W. Schreiner*, R. Karch*, F. Neumann*, M. Neumann**,
S. M. Rödler***, and A. End****

* Department of Medical Computer Sciences, University of Vienna, Vienna, Austria

** Institute of Experimental Physics, University of Vienna, Vienna, Austria

*** Department of Cardiology, University of Vienna, Vienna, Austria

**** Department of Cardiothoracic Surgery, University of Vienna, Vienna, Austria

Correspondence: Wolfgang.Schreiner@akh-wien.ac.at

Abstract. Models of coronary arterial trees are constructed by an optimization procedure. Starting from an initial tree, representing the gross anatomy of the coronary arteries, additional segments of the tree are grown in an optimized fashion to provide the smaller vessels representing a significant portion of the microcirculation. These models lend themselves as a realistic substrate for hemodynamic simulations, including intervention planning.

Keywords: Optimization, Arterial Tree Model, Vascular Growth

Introduction

It is generally assumed that arterial trees have been optimized for their task of transporting blood to the respective perfusion sites. Hence one can expect that a procedure involving "optimization" has the potential of inducing model structures closely related to what is found in reality [1][2][3]. This is the basic idea of constrained constructive optimization (CCO), a computational technique developed to generate optimized models of arterial trees from first principles. Without the input of anatomical data, CCO generates the structure of a tree of tubes by adding segment after segment in a geometrically optimized fashion.

In previous reports we have described in detail the method of CCO and the choice of physiological parameters [4], and analyzed branching angles, pressure profiles, segment radii, and perfusion inhomogeneity [5][4][6], most of which showed satisfactory agreement with experimental measurements [7][8].

Model Tree Construction

The concepts of CCO allow for a heuristic step-by-step construction of tree models, which is very simple and straightforward. Given a tree of N_{term} terminal segments, one more terminal can be added along the following lines:<

A new terminal site is selected randomly but with a conditional probability avoiding too close a vicinity to any of the existing tree segments [9]. The process of tossing is restricted to a "perfusion area" defined in mathematical terms so as to represent a piece of tissue to be supplied.

The new terminal site is connected to *any one* (say k) of the existing segments in the vicinity. The newly created bifurcation is moved around, enforcing after each movement the boundary conditions and recomputing the optimization target (e.g. intravascular volume V). Movements are then directed along the gradient [10][11] towards the minimum of V . This is called **geometric optimization**. The value V_k achieved for the (local) minimum when connecting to segment k is recorded in memory and the connection dissolved again.

All other segments within the vicinity are explored as possible connection sites in the same way, each time recording the value achieved for the target function. Finally, the connection with the "best local optimum" is adopted and made permanent. This is called **structural optimization** (or "connection search"). Note that geometric optimization is nested within structural optimization, and boundary conditions are reenforced many times during each process of geometric optimization.

Features of CCO Trees

The technical approach of generating optimized arterial tree models has previously been explained and evaluated in detail [12][13][4][6][14]. At each stage of development a CCO model satisfies a full set of physiologic constraints, incorporating a bifurcation law [7][15]

$$r_0^Y = r_1^Y + r_2^Y \quad (1)$$

and the following boundary conditions for pressures and flows (assuming constant blood viscosity η):

Each terminal segment provides a prescribed flow at the same pressure, intended to represent the physiologic need of a more or less uniform pressure to supply the deeper and more distal orders of the arterial tree not covered by the model.

Fully grown CCO models show some characteristic properties:

Terminal segments are distributed as evenly as possible over the perfusion area, imitating a homogeneous supply of blood. The residual heterogeneity compares well with experimental values.

At each step of growth the direction of development (e.g. the site where growth occurs) is influenced by the structure which already exists.

Structures developed in early stages of model growth are transmitted to become major vessels later on.

Regions with lower density of supply attract new terminals, thus imitating angiogenesis in real vascular trees induced by a lack of supply. The developing

arterial structure induces a residual spatial flow heterogeneity compatible with measurements.

CCO with Staged Growth

The original method of CCO generates arterial model trees of homogeneous structure where segments from the very early stages of construction turn into the main vessels of the final trees.

However, in some cases it is desirable to coin model trees upon real anatomy. For example, one may want the model to represent the anatomical coarse of the major coronary branches and side branches which all individuals have in common, or the model is intended to represent the left coronary artery bed of a particular patient. This can be achieved by specifying major branches initially (prior to CCO), e.g. by extracting real world coordinates and radii of the respective segments from an angiogram. The specification of such an "initial tree" bypasses the first few steps of CCO and substitute the random arrangement of the first few segments (which later on become the major segments).

CCO can then be started by implementing the boundary conditions in the initial tree and adding segment by segment in order to add microvasculature to the large segments. The result is a hybrid model, whose large arteries represent the externally supplied initial tree and whose smaller vessels form an optimized mesh accomplishing the distribution into the microcirculation.

In a recent study [16] we have investigated the basic idea, the feasibility and theoretical aspects of staged growth. Instead of anatomical data from angiography, small CCO models were used as "initial trees" as follows. CCO was performed within a sub-domain of the perfusion area to grow a tree consisting of a few segments ($N_{\text{term}}@ 100$) only. Then CCO was terminated, the perfusion domain enlarged and CCO resumed until the final number of terminals was reached. It was clearly evident how the structure of the initial tree influences the structure successively developing on top of it when comparing with a CCO tree grown from scratch over the entire domain (without any staging).

In an even more refined mode, multiple stages of growth can be used [16]. For example, the probability distribution for casting the new terminals can be changed over time in such a way that the segments at the epicardial surface (of a heart wall model) are generated first. By this simple modification of CCO the major vessels in the model are located close to the surface and thus mimic an additional and very important feature of real coronary trees. During completion of the model, the probability distribution of casting can be adapted so as to compensate the bias during the initial phase and end up with an even distribution of terminal sites over the entire volume. It is even possible to end up with a predefined gradient of perfusion density, say from epi- to endocardial layers [16]. Figure 1 shows the difference between pure CCO and staged growth over a slice of tissue representing a ventricular wall.

In formal terms, staging CCO represents an additional condition imposed. Optimization is not allowed to occur over the entire perfusion domain during the whole period of growth. While this modification of CCO successfully can induce more realistic structures of coronary models, it does not model the mechanisms themselves leading to these structures in reality : Real coronary arteries prevent being squeezed by intramyocardial pressure (IMP) since this would increase resistance and hamper supply. The gain in structural fidelity (and physiologic functionality) of the model with staged growth is achieved at the expense of a slightly enlarged intravascular volume (i.e. the very CCO target function).

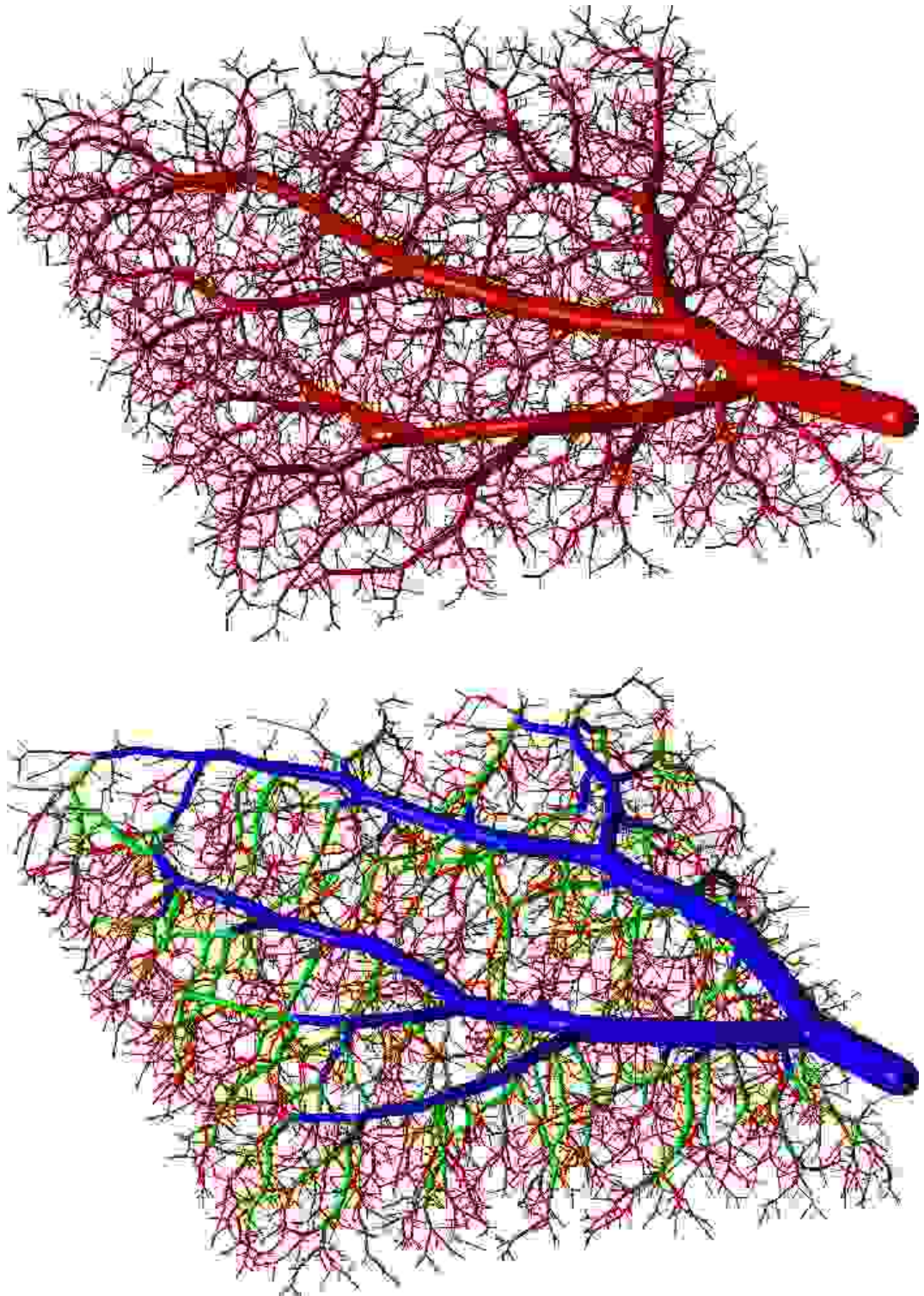


Figure 1. Visual representation of 3-dimensional staged growth modeling a coronary arterial bed. Panel (a): Reference tree grown by conventional CCO. Panel (b): Staged growth, starting at the upper surface of the perfusion volume and gradually extending towards the bottom. Large segments produced during the initial stage lie close to the upper surface and are shown in blue, main transmurals arteries are shown in green, small vessels in red. Visualization was performed by representing the vessel segments as the iso-surface of a pseudopotential assigned to the whole tree [17].

References

- [1] Cohn, D.L., "Optimal systems: II. The cardiovascular system", *Bulletin of Mathematical Biophysics*, vol. 17, pp. 219-227, 1955.
- [2] LaBarbera, M., "Principles of design of fluid transport systems in zoology", *Science*, vol. 249, pp. 992-999, 1990.
- [3] Thompson, D.W., "*On growth and form*", 2 Edition, Cambridge University Press, Cambridge, 1942.
- [4] Schreiner, W. and Buxbaum, P.F., "Computer-optimization of vascular trees", *IEEE Transactions on Biomedical Engineering*, vol. 40, pp. 482-491, 1993.
- [5] Bassingthwaite, J.B., King, R.B. and Roger, S.A., "Fractal nature of regional myocardial blood flow heterogeneity", *Circulation Research*, vol. 65, pp. 578-590, 1989.
- [6] Schreiner, W., Neumann, M., Neumann, F., Rödler, S.M., End, A., Buxbaum, P.F., Müller, M.R. and Spieckermann, P., "The branching angles in computer-generated optimized models of arterial trees", *Journal of General Physiology*, vol. 103, pp. 975-989, 1994.
- [7] Zamir, M. and Chee, H., "Segment analysis of human coronary arteries", *Blood Vessels*, vol. 24, pp. 76-84, 1987.
- [8] Zamir, M. and Chee, H., "Branching characteristics of human coronary arteries", *Canadian Journal of Physiology and Pharmacology*, vol. 64, pp. 661-668, 1986.
- [9] Schreiner, W. and Buxbaum, P.F., "Computer-optimization of vascular trees", *IEEE Transactions on Biomedical Engineering*, vol. 40, pp. 482-491, 1993.
- [10] Press, W.H., Teukolsky, S.A., Vetterling, W.T. and Flannery, B.P., "*Numerical recipes in FORTRAN, The art of scientific computing*", 2 Edition, Cambridge University Press, Cambridge, 1992.
- [11] Rabbitsch, G., Perktold, K. and Guggenberger, W., "Numerical analysis of intramural stresses and blood flow in arterial bifurcation models", *Computational Biomedicine*, eds. K.D.Held, C.A.Brebbia, R.D.Ciskowski & H.Power, Computational Mechanics Publications, Southampton Boston, pp. 149-156, 1993.
- [12] Karch, R., Neumann, F., Neumann, M. and Schreiner, W., "A three-dimensional model for arterial tree representation, generated by constrained constructive optimization", *Computers in Biology and Medicine*, vol. 29, pp. 19-38, 1999.
- [13] Schreiner, W., "Computer generation of complex arterial tree models", *Journal of Biomedical Engineering*, vol. 15, pp. 148-150, 1993.
- [14] Schreiner, W., Neumann, F., Karch, R., Neumann, M., Rödler, S.M. and End, A., "Shear stress distribution in arterial tree models, generated by constrained constructive optimization", *Journal of Theoretical Biology*, vol. 198, pp. 27-45, 1999.
- [15] Zamir, M., "Distributing and delivering vessels of the human heart", *Journal of General Physiology*, vol. 91, pp. 725-735, 1988.
- [16] Karch, R., Neumann, F., Neumann, M. and Schreiner, W., "Staged growth of optimized arterial model trees", *Ann Biomed Eng*, 2000 [in press].
- [17] Neumann, F., Neumann, M., Karch, R. and Schreiner, W., "Visualization of computer-generated arterial model trees", Chapter 24, *Simulation Modelling in Bioengineering*, eds. Cerrolaza, M., Jugo, D., and Brebbia, C. A., Computational Mechanics Publications, Southampton Boston, pp. 259-268, 1996.

:

