

Polynomial Decomposition for a P-wave Detection and Analysis

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Abstract. Aim of this study was to assess a polynomial decomposition for P-wave detection and analysis including classification by shape and parameters estimation. Recordings from PhysioNet database were used to get P-wave models and validate the proposed method. The normalized root mean squared error between each P-wave and its fitting was computed to select the best polynomial degree for positive, biphasic and notched models. Detection efficiency and right classification rate for proposed method were 99.86% and 91.25% (for the training set), and 99.02% and 90.66% (for the validation set). Results of the proposed method were compared to another automatic method (QTdb-method). The proposed method had lower errors for most indices (onset error proposed method vs. QTdb-method: 3.15 ± 1.42 samples vs. 4.27 ± 3.78 samples, $p < 0.01$; peak error proposed method vs. QTdb-method: 2.34 ± 1.86 samples vs. 2.98 ± 3.66 samples, ns; duration error proposed method vs. QTdb-method: 4.65 ± 2.57 samples vs. 5.36 ± 3.29 samples, $p < 0.05$; end errors proposed method vs. QTdb-method: 4.04 ± 3.09 samples vs. 3.49 ± 2.91 samples, $p < 0.01$). The preliminary results show that the proposed method can be used not only to detect and classify P-waves, but to robust estimate parameters of detected P-waves.

Keywords: Polynomial decomposition, P-wave detection and classification, parameters estimation

1. Introduction

The analysis of P-wave on the electrocardiogram (ECG) can be useful to detect abnormalities of the heart associated with the atria. Changes in the morphology of P-wave and its parameters are used in description of various pathologies of the atria, especially, to predict atrial fibrillation [Darbar et al., 2008 and Ishida et al., 2010]. However, many authors have reported difficulties in time-domain analysis because of the short duration and small amplitude of the P-wave. Therefore, it has been suggested to use ECG segmentation [Lepage et al., 2001] or model-based analysis [Couceiro et al., 2008] to describe atrial activity. Traditionally, methods of P-wave analysis are based on signal averaging to suppress Gaussian noise. Cross-correlation function-based algorithm, minimum square error-based algorithm and some others were suggested for P-wave alignment to improve quality of time-domain analysis and estimation of morphological parameters [Censi et al., 2006].

This paper presents the use of polynomial decomposition for robust (under the assumption of Gaussian noise) P-wave detection and classification with further parameters estimation for different types of P-wave.

2. Methods

2.1. Model parameter estimation

After basic preprocessing including filtering and QRS detection, a polynomial decomposition method is used to analyze each single P-wave on the ECG. It is known that P-waves can be classified into different classes [Catalano, 2002]. The most common types of shape are positive, biphasic and

notched. Positive is a normal rounded and upright P-wave, biphasic P-wave has two distinct phases and notched P-wave has two peaks.

Polynomial approximation for P-wave is obtained using the following model:

$$m_{iL} = c_{i0} + c_{i1}k + c_{i2}k^2 + \dots + c_{iL}k^L \quad (1)$$

where

m_{iL} = model of K samples
 i = type of P-wave shape
 c_{il} = approximation coefficients
 L = polynomial degree.

This equation can be transformed into the following parametric form:

$$m_{iL} = A_m \cdot \sum_{l=0}^L a_{il} (x \cdot k - k_0)^l \quad (2)$$

where

a_{il} = approximation coefficients
 A_m = P-wave amplitude measured as difference between maximum and minimum of P-wave
 x = P-wave duration measured as difference between onset and end of P-wave
 k_0 = peak location.

The following system of equations should be solved to find the coefficients of the approximating polynomial:

$$\begin{bmatrix} \sum_{k=1}^K k^{2L} & \sum_{k=1}^K k^{2L-1} & \dots & \sum_{k=1}^K k^L \\ \sum_{k=1}^K k^{2L-1} & \sum_{k=1}^K k^{2L-2} & \dots & \sum_{k=1}^K k^{L-1} \\ \dots & \dots & \dots & \dots \\ \sum_{k=1}^K k^L & \sum_{k=1}^K k^{L-1} & \dots & K \end{bmatrix} \times \begin{bmatrix} c_{iL} \\ c_{iL-1} \\ \dots \\ c_{i0} \end{bmatrix} = \begin{bmatrix} \sum_{k=1}^K k^L \cdot U_k \\ \sum_{k=1}^K k^{L-1} \cdot U_k \\ \dots \\ \sum_{k=1}^K U_k \end{bmatrix} \quad (3)$$

where

U_k = samples of the template for each considered type of P-wave.
 Templates are obtained by averaging P-waves of the same type.

2.2. P-wave detection and classification

Solving Eq. 3 for each type of P-wave shape (positive, biphasic and notched), the coefficients of the three types of P-wave are found. These models are then used to detect the P-wave: the following maximum likelihood criterion is used:

$$i = \arg \max_{i=1,3} \{ \Lambda_i(\bar{U}) \} \quad (4)$$

where

$$\begin{aligned} \bar{U} &= (U_1, U_2, \dots, U_K) = \text{set of P-wave samples.} \\ \Lambda_i(\bar{U}) &= \frac{w_i(\bar{U})}{w_0(\bar{U})} \end{aligned} \quad (5)$$

is the likelihood ratio (LHR) for i -type of P-wave, where

$$\begin{aligned} w_i(\bar{U}) &= \text{probability distribution for } i\text{-type of P-wave} \\ w_0(\bar{U}) &= \text{probability distribution for noise.} \end{aligned}$$

Inserting the expressions of a Gaussian distribution and white noise in Eq. 5 and after a few transformations:

$$\Lambda_i(\bar{U}) = \sum_{k=1}^K U_k \cdot m_{iL} - \frac{1}{2} \cdot \sum_{k=1}^K m_{iL}^2 \quad (6)$$

The maximum of LHR will correspond to the type of the model that is most similar to the analyzed signal. A value minor than zero indicates absence of a signal, i.e. absence of P-wave in the analyzed segment. If $\Lambda_1(\bar{U})$ is maximal then detected P-wave has positive shape, $\Lambda_2(\bar{U})$ for biphasic and $\Lambda_3(\bar{U})$ for notched.

After P-wave is detected and classified, parameters estimation is possible based on the calculation of the derivative of LHR:

$$\frac{d\left(\sum_{k=1}^K U_k \cdot m_{iL} - \frac{1}{2} \cdot \sum_{k=1}^K m_{iL}^2\right)}{d\lambda} = 0 \quad (7)$$

where

λ = is estimated parameter: A_m , x or k_0 from Eq. 2.

As an example, for $\lambda=A_m$ and replacing $\sum_{l=0}^L a_{il}(x \cdot k - k_0)^l$ with m_{iL}^* for Eq. 2 in Eq. 7:

$$\frac{d\left[A_m \sum_{k=1}^K U_k \cdot m_{iL}^* - \frac{1}{2} A_m^2 \cdot \sum_{k=1}^K (m_{iL}^*)^2\right]}{dA_m} = 0 \quad (8)$$

From Eq. 8 estimation for A_m equals to:

$$A_m = \frac{\sum_{k=1}^K U_k \cdot m_{iL}^*}{\sum_{k=1}^K (m_{iL}^*)^2} \quad (9)$$

Similarly, estimations for x and k_0 can be obtained that are as in the following equations:

$$\frac{d}{dx} \left[A_m \sum_{k=1}^K U_k \cdot \sum_{l=0}^L a_{il}(x \cdot k - k_0)^l - \frac{1}{2} A_m^2 \cdot \sum_{k=1}^K \left(\sum_{l=0}^L a_{il}(x \cdot k - k_0)^l \right)^2 \right] = 0 \quad (10)$$

$$\frac{d}{dk_0} \left[A_m \sum_{k=1}^K U_k \cdot \sum_{l=0}^L a_{il}(x \cdot k - k_0)^l - \frac{1}{2} A_m^2 \cdot \sum_{k=1}^K \left(\sum_{l=0}^L a_{il}(x \cdot k - k_0)^l \right)^2 \right] = 0 \quad (11)$$

Real algorithm finds duration x first. Method of successive approximations is used because of the complexity of differential equations (Eq. 9, 10 and 11). Thus, the first and the last items of the model are estimated onset and end of the P-wave, respectively. Then k_0 and A_m can be calculated using Eq. 9 and Eq. 11.

3. Data

Two databases were used in this study. The first one is the T-Wave Alternans Challenge Database, made available for the PhysioNet/Computers in Cardiology Challenge 2008 [Moody, 2008]. It contains 100 2-, 3-, and 12-lead ECG records sampled at 500 Hz with 16-bit resolution over a ± 32 mV range, including subjects with risk factors for sudden cardiac death as well as healthy controls and synthetic cases with calibrated amounts of T-wave alternans. Fifty-six two-minute recordings of lead I were extracted from this database. One third of the recordings formed the training set. The remaining signals were used to validate the method of detection and classification (validation set). Different types of P-waves were present: in particular, 42 recordings were visually identified having mostly positive P-waves, 6 biphasic and 8 notched.

The second used database is the QT-database of PhysioNet [Laguna et al., 1997], consisting of over than 100 fifteen-minute two-lead ECG recordings. In each recording, from 30 to 50 beats are labeled with onset, peak, and end markers for P, QRS, T, and (where present) U waves. Ninety recordings from this database formed the testing set consisting of positive P-waves. They were used to compare estimation results of the proposed method with automatic method based on threshold method applied to a differentiated ECG [Laguna et al., 1994] and manual values.

4. Results

4.1. Polynomial degree selection

Polynomial degree of Eq. 3 is unknown and was chosen based on the normalized root-mean-square error (NRMSE). Briefly, P-waves were manually detected, synchronized and then averaged for each of the training set signal. This averaged P-wave was used to get the model for each signal. Models of the same type were normalized to the amplitude of 0.2mV and duration of 128ms. Figure 1 shows how the increase of the polynomial degree decreases NRMSE for the group of positive type.

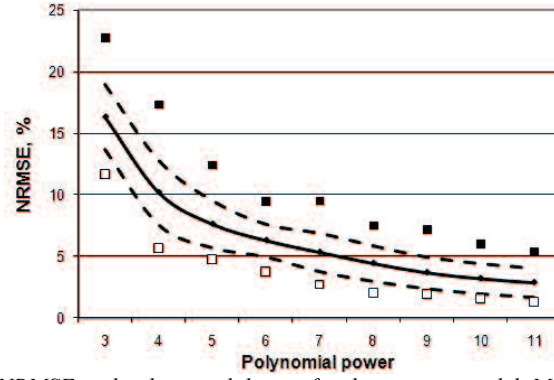


Figure 1. Relation between NRMSE and polynomial degree for the positive model. Mean value (solid line), standard deviation (dotted line), maximum (black squares) and minimum (white squares) values.

Defining as acceptable a NRSME minor than 10%, a degree of 6 was selected for positive type and a degree of 7 for notched and biphasic. Using higher degree polynomials did not improve the modeling accuracy significantly, but complicated calculations.

4.2. Training set

Table 1 shows the estimated polynomial coefficients for the different types of the P-wave and Figure 2 shows P-wave's models described by these coefficients.

Table 1. Polynomial coefficients for different models.

Coefficient	Positive model	Biphasic model	Notched model
c_0	-5.9788	0.4411	-10.6057
c_1	12.7686	4.6776	15.8182
c_2	-1.8931	-0.7467	-1.4233
c_3	0.1346	0.0567	-0.0093
c_4	-0.0038	-0.0038	0.0039
c_5	$4.60 \cdot 10^{-5}$	$1.00 \cdot 10^{-4}$	$-1.22 \cdot 10^{-4}$
c_6	$-1.91 \cdot 10^{-7}$	$-1.96 \cdot 10^{-6}$	$1.39 \cdot 10^{-6}$
c_7	-	$1.05 \cdot 10^{-8}$	$-5.35 \cdot 10^{-9}$

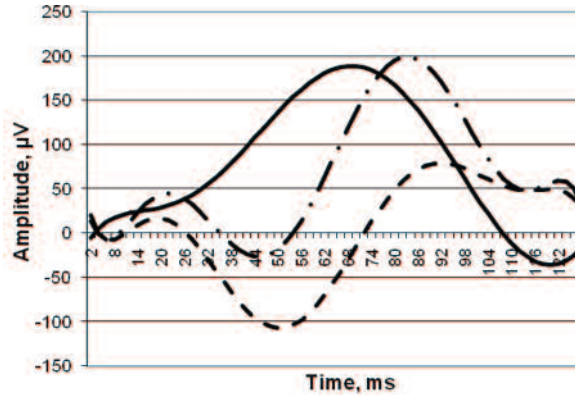


Figure 2. P-wave models: positive (solid line), biphasic (dashed), notched (dash-and-dotted).

4.3. Validation set

Table 2 presents likelihood ratios calculated using Eq. 6 for P-waves with different types from the validation set. It can be noted that maximum LHR corresponds to the type of model similar to real signal.

Table 2. Likelihood ratios for different models.

Signal	Positive model	Biphasic model	Notched model
Positive	0.21±0.03	-0.06±0.03	0.17±0.04
Biphasic	-0.28±0.04	0.03±0.01	-0.11±0.04
Notched	-0.04±0.03	0.03±0.01	0.10±0.03
No P-wave	-0.21±0.09	-0.07±0.04	-0.18±0.08

Table 3 reports the detection efficiency and right classification rate of P-waves by their types for both the training and validation sets.

Table 3. Detection and classification rates.

	Total # of segments	Detection efficiency	Right classification
Training set			
Positive	1240	99.76%	85.37%
Biphasic	357	100.00%	100.00%
Notched	532	100.00%	99.06%
Total	2129	99.86%	91.25%
Validation set			
Positive	2771	99.78%	86.00%
Biphasic	718	94.99%	100.00%
Notched	880	99.89%	98.07%
Total	4369	99.02%	90.66%

4.4. Testing set

To assess the estimation performance, a comparison with another automatic method was performed. Using manually annotated recordings from the testing set, errors for proposed method were computed and compared to another existing automatic method. Results are shown in Table 4.

Table 4. Methods comparison (* $p < 0.05$, § $p < 0.01$ paired t-test).

Parameter	QTdb-method	Proposed method
Detection efficiency, %	91.68±19.86	92.68±11.70
Onset error, samples	4.27±3.78	3.15±1.42 §
End error, samples	3.49±2.91	4.04±3.09 §
Peak error, samples	2.98±3.66	2.34±1.86
Duration error, samples	5.36±3.29	4.65±2.57 *

It can be noted that detection efficiency is higher and onset, peak and duration errors are lower for the proposed method. The only parameter where proposed method performed worse is the end error.

From Table 4 it can be also noted that standard deviation for the proposed method is lower for all parameters except end error. Results of t-test for standard deviation for the whole testing set are given in Table 5. The lower standard deviation for the proposed method underlines that more stable results are obtained

Table 5. T-test results for standard deviation.

Parameter	P-value	Decision
Onset error, samples	0.019	Significant
End error, samples	0.436	No difference
Peak error, samples	0.035	Significant
Duration error, samples	0.004	Significant

5. Conclusions

The preliminary results of the proposed methods show that it can be used not only to detect and classify P-waves, but to robustly estimate parameters of detected P-waves. The limitation of presented study is the manual localization of the segment, where the proposed method is then applied. A larger number of P-wave models also need to be generated, for example using other leads in addition to lead I and including negative shaped models.

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